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UE-6001

First Year B. E. (Sem. I) Examination

April/May – 2012

Engineering Mathematics : Paper - I

Time : Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ F. Y. B. E. (SEM. 1)

Name of the Subject :

☛ ENGINEERING MATHEMATICS : PAPER - 1

☛ Subject Code No. : 6 0 0 1 ☛ Section No. (1, 2,.....) : Nil

Seat No. :

Student's Signature

(2) All questions are compulsory.
(3) Figure on the right indicate marks.
(4) Draw the figure whenevr it is necessary.

1 a) Do as directed. **[10]**

1. State Maclaurins theorem for one variable.
2. If $y = e^{2x}$ then obtain y_n.
3. Evaluate $\lim_{x\to\infty} \frac{e^x - 1}{x^4}$.
4. Define relation between circular & hyperbolic functions.
5. Prove that $1 - \tanh^2 x = \sec h^2 x$.

b) State & Prove Leibnitz's theorem for the nth derivative of the product of two functions. **[04]**

c) Attempt the following. **[06]**

1. Expand $\sin x$ in powers of $\left(x - \frac{\pi}{4}\right)$ Hence find the value of $\sin 46^0$.
2. By using Taylor's series, arrange $\left[7 + (x+2) + 3(x+2)^2 + (x+2)^4\right]$ in powers of x.

2 a) Find radius of curvature of the curve $(x^2 + y^2)^2 = a^2(x^2 - y^2)$ at any point where tangent is parallel to X-axis. **[05]**

b) If $y = \sin\left(m \sin^{-1} x\right)$ then prove that **[04]**

$$\left(1 - x^2\right) y_{n+2} = (2n+1) x y_{n+1} + \left(n^2 - m^2\right) y_n .$$

c) **Attempt any two of the following** **[06]**

1. $\lim_{x\to 1}\dfrac{x^x - x}{x - 1 - \log x}$

2. $\lim_{x\to 0}\dfrac{1-\cos x+\sin x+\log(1-x)}{x\tan x\left(e^x-1\right)}$

3. $\lim_{x\to 0}\left(\dfrac{1}{x^2}-\cot^2 x\right)$

3 a) State & Prove De Moiver's theorem. **[04]**

b) **Attempt any two of the following.** **[08]**

1. If α and β are two roots of the equation $x^2-2x+4=0$ then prove that $\alpha^n+\beta^n=1$.

2. Prove that $\tanh^{-1}(\cos\theta)=\cosh^{-1}(\cos ec\theta)$

3. If $i^{\alpha+i\beta}=\alpha+i\beta$ then prove that $\alpha^2+\beta^2=e^{-(4n+1)\pi\beta}$.

c) Prove that $\tanh^{-1}x=\dfrac{1}{2}\log\dfrac{1+x}{1-x}$; where $|x|<1$. **[03]**

Section-II

4 a) **Do as directed.** **[10]**

1. Define Linear differential equation of first order.
2. Give formula for radius of curvature in polar form.
3. Define order and degree of the differential equation.
4. Sketch the polar curve $r=a(1-\cos\theta)$.
5. Define Asymptote.

b) **Attempt the following** **[05]**

1. Trace the curve $\left(x^2-a^2\right)\left(y^2-b^2\right)=a^2b^2$

c) **Attempt any one of the following.** **[04]**

1. Find the points on the parabola $y^2=8x$ at which the radius of curvature is $\dfrac{125}{16}$.

2. Find the perimeter of the closed loop; of the curve $9ay^2=x(x-3a)^2$.

5 a) **Solve any three of the following.** **[09]**

1. $x^2\dfrac{dy}{dx}+xy+\sqrt{1-x^2y^2}=0$

2. $(\tan y+x)dx+\left(x\sec^2 y-3y\right)dy=0$

3. $\dfrac{dy}{dx}=\dfrac{y}{x}+\tan\left(\dfrac{y}{x}\right)$

4. $\dfrac{dy}{dx}=\dfrac{y^3}{e^{2x}+y^2}$ **[06]**

b) Attempt any two of the following

1. Solve $xp^2 - 2py - 4x = 0$.
2. Solve $x = y + p^2$.
3. Solve $(px - y)(x - py) = 2p$ using the transformation $x^2 = u, y^2 = v$.

6 Attempt any two of the following **[16]**

1. State and formulate SIS-model for spread of epidemic disease. Also obtain its solution with its interpretation.
2. State and explain LR-series circuit electrical network model. Obtain its solution by considering EMF source is given and interpret the solution.
3. A simple electric circuit contains a resistance 10 ohms and an inductance of 4 henries in series with an induced e.m.f of $100\sin 200t$ volts. If the current $i = 0$ when the time $t = 0$, find the current when t=0.001.